Modeling neural flow through linearization procedures

Radu Constantinescu , Carmen Ionescu, Mihai Stoicescu
Department of Physics, University of Craiova

1. Neuron models: from real world to mathematical and technical models.
2. Membrane phenomena in neural cells.
3. Cable equation of the axon.
4. Neural networks. Hopfield type models.

Models for neurons

From real world to mathematical and technical models

y models?
- Modeling – the natural mental action in understanding the world
- The model should be realistic
- It has to capture the biological complexity but to allow further stigations.

dels may describe an object or a phenomenon as:
- Structure (mechanical model)
- Function (electronic or mathematical or computer model)
- Evolution (transport through membrane or along axon, synaptic transmission)

main neuron models are expressed in terms of
- Mathematical equations (Hodgkin-Huxley equations)
- An imaginary construction following the laws of physics (Eccles model)

Membrane phenomena in neural cells

tric conduction throught membrane

e energy spent as work of electrical force to move an electric charge along electric circuit is given by:

$$W_e = Q(\Phi_0 - \Phi_i) = zF(\Phi_0 - \Phi_i) \quad (1)$$

Φ_0 ; Φ_i = extra and intracellular potentials

z = valence of the ions

F = Faraday's constant [9.649×10^4 C/mol]

nsport phenomena through membrane

According to Ohm's law, current density and electric field are related by

$$\overline{J} = \sigma \overline{E} = -\sigma \nabla \Phi \quad (2)$$

where σ is the conductivity of the medium

r a system with many types of ions:

$$\sigma = \sum u_k \cdot \frac{z_k}{|\ |} c_k \quad (3)$$

where:

u_k = ionic mobility [$cm^2/(V \cdot s)$]

z_k = valence of the ion

ording to Fick’s Law:

$$\bar{j}_{kD} = -D_k \nabla c_k$$
$$D_k = \frac{u_k R T}{|z_k| F}$$

where:

T = absolute temperature [K]

R = gas constant [8.314 J/(mol·K)]

J_{kD} = ionic flux (due to diffusion) [mol/(cm²·s)]

D_k = Fick's constant (diffusion constant) [cm²/s]

c_k = ion concentration [mol/cm³]

$c_{i,k}$ = intracellular concentration of the kth ion

$c_{o,k}$ = extracellular concentration of the kth ion

st – Plank Equation for neuron:

c flux

$$\bar{j}_{kD} + \bar{j}_{ke} = -D_k \left(\nabla c_k + \frac{c_k z_k F}{R T} \nabla \Phi \right)$$

tric current density

$$-D_k z_k F \left(\nabla c_k + \frac{c_k z_k F}{R T} \nabla \Phi \right)$$

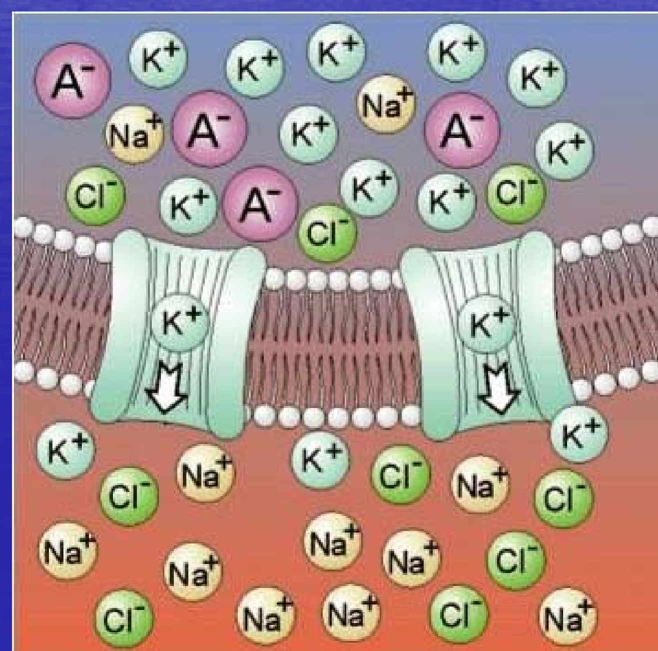
brium condition for current density:

$$\vec{J}_k = 0 \Rightarrow$$

$$\nabla c_k = -\frac{c_k z_k F}{RT} \nabla \phi$$

$$\ldots_k = -\frac{z_k F}{R T} (\Phi_o - \Phi_i)$$

$$V_k = -\frac{R T}{F} \ln \frac{c_{i,k}}{\ldots}$$



ivalent electric circuit for neural cell

neuron membrane behaves as a capacitor with many conducting nels joining its two sides, the extracellular and the cytoplasmic ones.

equivalent electric circuit is presented below and the Kirchhoff laws

$\nabla \vec{j} = 0$ =>

$$\sum_{k=1}^{n} \varepsilon_k I_k = 0$$

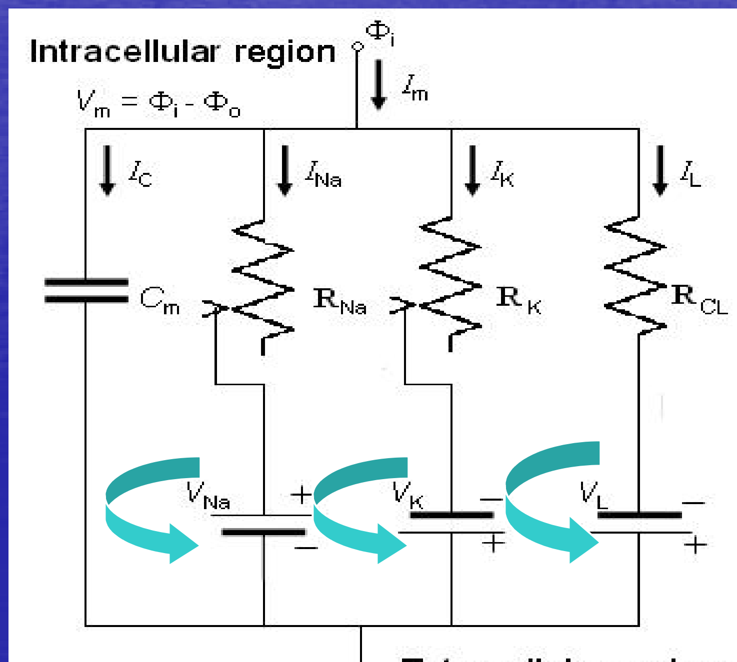
$\frac{1}{\sigma}\vec{j} = \vec{E} \Rightarrow$

$$\sum_{k=1}^{n} \varepsilon_k R_k I_k = \sum_{k=1}^{m} U_k$$

ith the concrete notations:

$I_c + I_{Na} + I_K + I_L = 0$

$$\frac{dV}{dt} = -g_{Na}(V - V_{Na}) - g_K(V - V_K) - g_L(V - V_L) + I_m$$

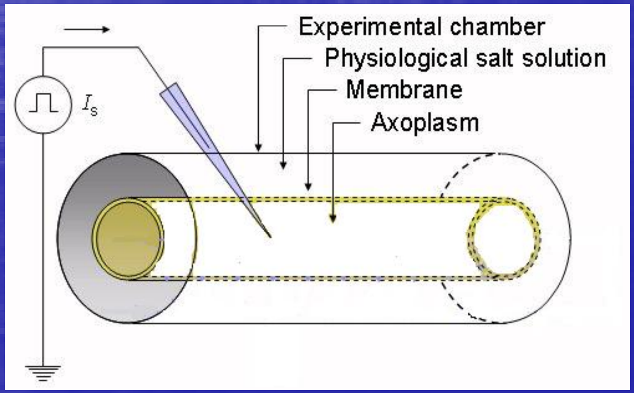


Cable equation for the axon

uppositions:

- the voltage depends on the point where is calculated
- the flow travel with constant velocity and the pulse maintains its riginal form during the propagation.

$$U = U(x,t)$$



onclusion: the dynamics obeys the wave equation and the

- The propagation equation has the form: $$\frac{\partial^2 U}{\partial x^2} = \frac{1}{\Theta^2}\frac{\partial^2 U}{\partial^2 t}$$

- Suppose the variation of the potential along the axon satisfies the relation:

$$\frac{\partial U}{\partial x} = I_i r_i + I_0 r_0$$

With new derivation of the previous relation we get:

$$\frac{\partial^2 U}{\partial x^2} = r_i \frac{\partial I_i}{\partial x} + r_0 \frac{\partial I_0}{\partial x}$$

The conservation of total charge imposes $$\frac{\partial I_i}{\partial x} = \frac{\partial I_0}{\partial x} = \upsilon_m$$

- Substituting this last equation in the wave equation, we obtain

$$(r_i + r_0)\upsilon_m = \frac{1}{\ }\frac{\partial^2 U}{\ } = \frac{\partial^2 U}{\ }$$

Neural networks. Hopfield type models

- oposed in 1982 by John Hopfield to explain the functioning of the mory as an asynchronous neural network.
- onsists in *n* totally coupled units, that is, each unit is connected to all er units except itself and change information recursively between m.
- he individual units are randomly updated, preserving their individual tes in the interval between two updates.
- uppositions:
  - o synchronization requirements (not a universal time is needed)
  - he neurons are not updated simultaneously
  - ymmetric transfer of information among units

The starting point is the Hodgkin-Huxley model and the Kirchhoff law:

$$C_m \frac{dV}{dt} = -g_{Na}(V - V_{Na}) - g_K(V - V_K) - g_L(V - V_L) + I_m$$

Suppositions:

- the potential is constant along the axon.
- in the same unit area, there is a delay between the voltage U on the esistive channels and the capacitive potential V.

The cable equation of the axon can be written as:

$$C_m \frac{dV}{dt} = -f(V, U, t) + I_m(t)$$

$$\frac{dU}{dt} = g(V, U, t); g(V, U, t) \xrightarrow{U \to V} 0$$

The previous system gives an expression of the voltage *V* in terms of *U.*

Mathematical form of the binary model

Experiments shown that there are two branches:

- a mostly liner branch

- a second nonlinear one

The two branches can be separated by using a binary variable S=+1 or -1.

From the previous consideration we conclude that the functioning of one neuron can be described in good agreement with the real neuron behavior by the equation:

$$\frac{dU}{dt} = a - b \cdot U(t) + c(1-S)I_m(t)$$

For many neurons, the Hopfield model proposed the equations:

$$\dot{u}_k(t) = -b_k u_k + \sum^{n} w_{kj} G_j(u_j) + v_k(t), \quad k = 1, \dots, n$$

merical simulations for the action potential

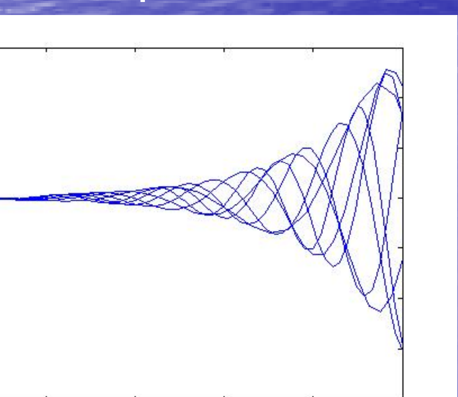
ing back to the cable equation and passing to the "wave" coordina $\tau = t - \frac{x}{c}$

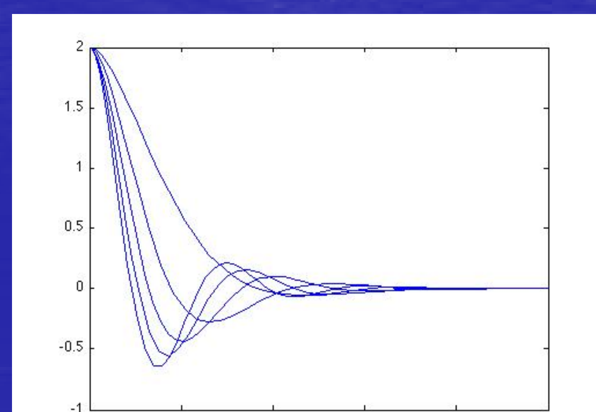
previous equation becomes:

$$U^{''}\text{-}AU^{'}\text{-}BU = 0$$

re: $A = \frac{\tau}{\lambda^2}; B = \frac{1}{\lambda^2}$

studied numerically the behavior of the voltage along the wave, and the ts are presented in the figure below:





Conclusions:

The propagation of neural flow and across neurons is a not yet
cidated problem.

There are many models from very complex as Hodgkin-Huxley,
tinuing with integrate or fire models, till very simple ones as binary
ofield-type models.

The reduction of the complexity of a model has to allow detailed
dies on network characteristics but not to spoil its biological content.

Using Hopfield model, we described the flow in terms of a simple

Thank you !